Economics 3P90 Fall 2012
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Using the Logit Model

Consider a logit model given by

$$P(Y_i = 1|X_{2i}, X_{3i}, ..., X_{ki}) = \frac{1}{1 + \exp(-(\beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + ... + \beta_k X_{ki}))} \quad (1)$$

In class, we saw how to use the Logit model to generate forecasts, and to assess the marginal effects of the regressors $X_{2i}, ..., X_{ki}$ on the conditional probability $P(Y_i = 1|X_{2i}, X_{3i}, ..., X_{ki})$, the odds ratio, and the log-odds ratio.

(a) **Forecasting Probability**

Given coefficient estimates $\hat{\beta}_1, ..., \hat{\beta}_k$, the predicted probability that $Y_i = 1$ given $X_{2i}, ..., X_{ki}$ is

$$\hat{P}(Y_i = 1|X_{2i}, X_{3i}, ..., X_{ki}) = \frac{1}{1 + \exp(-(\hat{\beta}_1 + \hat{\beta}_2 X_{2i} + \hat{\beta}_3 X_{3i} + ... + \hat{\beta}_k X_{ki}))} \quad (2)$$

(b) **Change in Log-Odds**

Given coefficient estimates $\hat{\beta}_1, ..., \hat{\beta}_k$, the change in the log-odds ratio $Z_i = ln(P_i/(1-P_i)$ per unit change in X_j is given by

$$\frac{\delta Z_i}{\delta X_j} = \hat{\beta}_j \quad (3)$$

(c) **Change in Odds**

Given coefficient estimates $\hat{\beta}_1, ..., \hat{\beta}_k$, the change in the odds ratio $P_i/(1 - P_i)$ per unit change in X_j is given by

$$\frac{\delta P_i/(1 - P_i)}{\delta X_j} = \exp(\hat{\beta}_j) \quad (4)$$

(d) **Percent Change in Odds**

Given coefficient estimates $\hat{\beta}_1, ..., \hat{\beta}_k$, the *percent* change in the odds ratio $P_i/(1 - P_i)$ per unit change in X_j is approximately given by

$$\frac{\%\Delta P_i/(1 - P_i)}{\Delta X_j} \approx \exp(\hat{\beta}_j) - 1 \quad (5)$$

(e) **Change in Probability**

Given coefficient estimates $\hat{\beta}_1, ..., \hat{\beta}_k$, the change in probability $\hat{P}_i$ per unit change in X_j is given by

$$\frac{\delta \hat{P}_i}{\delta X_j} = \hat{\beta}_j (1 - \hat{P}_i)\hat{P}_i \quad (6)$$