Economics 3P90 Fall 2012
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Hypothesis Testing: The t-test

Suppose that we're estimating the following regression model:

$$Y_i = \beta_1 + \beta_2 X_i + u_i \tag{1}$$

and wish to test either a two-tailed (two-sided) hypothesis:

$$H_0 : \beta_1 = \beta_1^* \tag{2}$$
$$H_1 : \beta_1 \neq \beta_1^* \tag{3}$$

or a one-tailed (one-sided) hypothesis:

$$H_0 : \beta_1 = \beta_1^* \tag{4}$$
$$H_1 : \beta_1 > \beta_1^* \tag{5}$$

using the OLS estimates $\hat{\beta}_1$.

Steps to carry out the t-test

1. Use the OLS formulas to obtain the estimate $\hat{\beta}_1$ and the sampling variance $v\hat{a}r(\hat{\beta}_1)$
2. Choose the desired level of significance α (or level of confidence $(1-\alpha)$)
3. Calculate the test statistic:
$$t = \frac{\hat{\beta}_1 - \beta_1^*}{\sqrt{v\hat{a}r(\hat{\beta}_1)}} \sim t_{n-2} \tag{6}$$
4. Find the corresponding critical value $t_{n-2,\alpha}$ **for one-sided hypothesis**, and $t_{n-2,\alpha/2}$ **for two-sided hypothesis**
5. Make a rejection decision as follows:

Type of Hypothesis	H_0 : **The Null**	H_1: **The Alternative**	**Reject** H_0: **if**
Two-tail	$\beta_1 = \beta_1^*$	$\beta_1 \neq \beta_1^*$	$\lvert t\rvert > t_{n-2,\alpha/2}$
Right-tail	$\beta_1 = \beta_1^*$	$\beta_1 > \beta_1^*$	$t > t_{n-2,\alpha}$
Left-tail	$\beta_1 = \beta_1^*$	$\beta_1 < \beta_1^*$	$t < -t_{n-2,\alpha}$

Refer to section 5.7 of the Gujarati text for additional details.